

# M-Theory Inflation from Multi M5-Brane Dynamics

Katrin Becker<sup>a,1</sup>, Melanie Becker<sup>b,2</sup> and Axel Krause<sup>b,3</sup>

<sup>a</sup> *Department of Physics, University of Utah,  
Salt Lake City, UT 84112-0830, USA*

<sup>b</sup> *Department of Physics, University of Maryland,  
College Park, MD 20742, USA*

## Abstract

We derive inflation from M-theory on  $S^1/Z_2$  via the non-perturbative dynamics of  $N$  M5-branes. The open membrane instanton interactions between the M5-branes give rise to exponential potentials which are too steep for inflation individually but lead to inflation when combined together. The resulting type of inflation, known as assisted inflation, facilitates considerably the requirement of having all moduli, except the inflaton, stabilized at the beginning of inflation. During inflation the distances between the M5-branes, which correspond to the inflatons, grow until they reach the size of the  $S^1/Z_2$  orbifold. At this stage the M5-branes will reheat the universe by dissolving into the boundaries through small instanton transitions. Further flux and non-perturbative contributions become important at this late stage, bringing inflation to an end and stabilizing the moduli. We find that with moderate values for  $N$ , one obtains both a sufficient amount of e-foldings and the right size for the spectral index.

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<sup>1</sup>katrin@physics.utah.edu

<sup>2</sup>melanieb@physics.umd.edu

<sup>3</sup>krause@physics.umd.edu

# 1 Introduction and Summary

Based on the recent progress of constructing de Sitter vacua in string-theory [1], [2] and M-theory [3], [4] there has been an extensive effort to derive cosmic inflation from string- and M-theory [5]<sup>4</sup>. Most of these approaches aim to derive a sufficiently flat potential to realize new inflation [8]. In these studies it became clear that in most cases (if not all) a considerable degree of fine-tuning has to be applied to achieve this goal. For a status report on the progress see [9]. Closely related to the problem of obtaining a single extremely flat slow roll direction, there is a second problem – the one of moduli stabilization. In order to have the universe rolling along an extremely flat direction in moduli space one has to ensure that the potential in all remaining directions is sufficiently curved upwards. In other words, all moduli except the modulus which serves as the inflaton have to be stabilized before the inflationary phase.

It is the goal of this paper to show that inflation can be naturally realized in M-theory. The key for this will come from the dynamics of many M5-branes<sup>5</sup>. Their mutual interactions, stemming from open membrane instantons, lead to exponential potentials. These potentials are too steep to give rise to inflation individually. However, when taken together, they increase the Hubble friction and lead to a specific type of inflation, known as assisted inflation [11]. It is therefore essential to have many M5-branes present and not just one. The great advantage of realizing inflation in M-theory in this way, is that there is no need to stabilize all moduli before the inflationary phase. This comes from the fact that the inflatons, which correspond to the distances between the M5-branes in the  $S^1/Z_2$  direction, turn out to correspond to the steepest possible directions of the potential (see fig.1). This is in striking contrast to a realization of inflation as new inflation. The assisted type of inflation, which is a generalization of power-law inflation [12], has only one parameter  $p$  which will be a function of  $N$ , the number of M5-branes being present. As we will see, both the number of e-foldings and the spectral index, which characterizes the power law spectrum of primordial curvature perturbations, lie in the right regime if we assume values around  $N \simeq 89$ . Moreover, we will find a natural exit from inflation and a reheating mechanism. The distances between the M5-branes, which play the role of the inflatons, grow during the inflationary phase. Once they have grown to a size comparable to the orbifold size itself, other flux and non-perturbative contributions to

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<sup>4</sup>The new recent proposal of obtaining inflation and standard cosmology from ghost dynamics [6] has so far not been derived from string-theory but seems promising as the ghost has to be an axion [7].

<sup>5</sup>The cosmological implication of a single M5-brane has been studied e.g. in [10].

the potential become important and terminate inflation. These other contributions, most notably gaugino condensation on the hidden boundary and boundary-boundary open membrane instantons, will stabilize the moduli through mechanisms described in [3]. Reheating occurs when the M5-branes collide with the boundaries. In particular the M5-branes colliding with the visible boundary will reheat our universe when they dissolve with the boundary via small instanton transitions [13], [14].

Let us motivate and provide some background for our approach. To break supersymmetry spontaneously one can either use fluxes or non-perturbative effects. The latter lead to exponential potentials. In the context of inflation, it has been known for a long time that simple exponential potentials of the form

$$U(\varphi) = U_0 e^{-\sqrt{\frac{2}{p}} \frac{\varphi}{M_{Pl}}} , \quad (1.1)$$

with a parameter  $p > 1$ , lead to power-law inflation [12] (by  $M_{Pl}$  we denote the reduced Planck-scale). In these models the scale-factor of the four-dimensional Friedmann-Robertson-Walker (FRW) universe grows with cosmic time  $t$  like

$$a(t) = a_0 t^p , \quad (1.2)$$

while the inflaton  $\varphi$  evolves as

$$\varphi(t) = \sqrt{2p} M_{Pl} \ln \left( \sqrt{\frac{U_0}{p(3p-1)}} \frac{t}{M_{Pl}} \right) . \quad (1.3)$$

This exact solution is valid for parameters  $p > 1/3$ . The two slow-roll parameters

$$\epsilon = \frac{M_{Pl}^2}{2} \left( \frac{U'}{U} \right)^2 , \quad \eta = M_{Pl}^2 \frac{U''}{U} , \quad (1.4)$$

where a prime indicates the derivative w.r.t. the inflaton  $\varphi$ , turn out to be constant

$$\epsilon = \frac{1}{p} , \quad \eta = \frac{2}{p} . \quad (1.5)$$

To obtain inflation, it is enough to demand that  $p > 1$  which guarantees that  $\ddot{a}(t) > 0$ . If in addition both slow-roll parameters should be sufficiently small to meet observational constraints, we will rather have to impose that  $p \simeq 100$ , as we will see later.

In this simple example the slow-roll parameters are constant and there is thus no exit from power-law inflation. We shall see that when embedded into string- or M-theory this presents, however, no problem as there are additional contributions which become relevant

eventually. These will modify the simple exponential potential and cause inflation to end. To the reader who associates inflation mostly with new inflation, let us remark that a period of inflation is characterized very broadly by the requirement,

$$\ddot{a}(t) > 0 , \quad (1.6)$$

which allows many more realizations than just having an extremely flat potential as would be required for new inflation. In particular, we will see that inflation can also be realized with very steep directions.

In heterotic M-theory [15], on which we want to focus subsequently, exponential potentials arise from open membrane instantons [16], [17], [18], [19] which wrap genus zero holomorphic 2-cycles on the Calabi-Yau manifold<sup>6</sup>. An open membrane instanton stretching between both boundaries (i.e. the  $\mathbf{Z}_2$  fixed planes) leads to a superpotential

$$W = h e^{-T}, \quad (1.7)$$

with  $T$  the Kähler-modulus whose real part measures the size of the 3-cycle covered by the open membrane instanton. In the large volume limit this will lead, apart from power-law corrections, to an exponential potential for the real part of  $T$  with parameter  $p = 1$  (a factor  $\sqrt{2}$  comes from the different normalizations of the kinetic terms for a complex and a real field). With this value of  $p$  there would be no inflation. Moreover, the inflaton  $\varphi$ , which we have used before to describe power-law inflation, has canonically normalized kinetic terms which is not the case for  $T$ . Upon transforming the real part of  $T$  to a canonically normalized field  $\varphi_T = M_{Pl} \sqrt{\frac{3}{2}} \ln(T + \bar{T})$ , we would end up with a double exponential instead of a simple exponential potential. This double exponential potential is again too steep to lead to inflation. It is therefore not possible to generate power-law inflation from just the boundary-boundary interaction arising from a single open membrane instanton.

There is, however, a very interesting multi-scalar extension of the power-law inflation scenario, called assisted inflation [11], which can give inflation even though the individual single-field potentials cannot. This inflation scenario is based on  $N$  scalar fields  $\varphi_i$ ,  $i = 1, \dots, N$ , each of which possesses a potential

$$U = U_0 e^{-\sqrt{\frac{2}{p}} \frac{\varphi_i}{M_{Pl}}} , \quad \forall i = 1, \dots, N . \quad (1.8)$$

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<sup>6</sup>Corrections to F-terms coming from higher genus instantons vanish due to holomorphy. We would like to thank E. Witten for pointing this out to us.

One can map this multi-field problem to the single field power-law problem and show that it leads as well to a power-law solution for  $a(t) = a_0 t^{p(N)}$ , where now [11]

$$p \rightarrow p(N) = Np. \quad (1.9)$$

This solution is valid if  $p(N) > 1/3$  and leads to inflation for  $p(N) > 1$ . Hence, even if the single exponential contributions are too steep to support inflation individually, nevertheless one can obtain inflation by choosing  $N$  large enough. We will see that this type of inflation arises naturally in the large volume limit from the dynamics of  $N$  M5-branes distributed along the  $\mathbf{S}^1/\mathbf{Z}_2$  orbifold interval.

We want to stress that there is one very important benefit when inflation is realized in string- or M-theory through assisted inflation. This benefit concerns the issue of moduli stabilization. We will see that the canonically normalized inflaton fields  $\varphi_i$  originate from the real parts of moduli  $Y_{ji} = Y_j - Y_i$ ,  $i, j = 1, \dots, N$  which describe the position difference of the  $N$  M5-branes along the  $\mathbf{S}^1/\mathbf{Z}_2$  interval. It is the  $Y_{ji}$  directions in which the potential decreases fastest. During the inflationary period these are the only directions where the potential falls off exponentially fast,  $U \sim e^{-Y_{ji} - \bar{Y}_{ji}}$ . Hence, this alleviates considerably the task to have all other moduli, except the inflatons, stabilized before the inflationary phase. We are no longer dealing with an extremely flat direction but instead with the opposite extreme, the steepest possible directions (see fig.1). To ensure that the universe is indeed following this steepest path, it is enough to require that all other directions are less steep than this exponential decrease. This is, however, a condition which is much easier met and could also allow for a mild runaway in the non-inflating directions.

The organization of this paper is as follows. In section 2 we explain the heterotic M-theory setup and derive the potential with  $N$  parallel M5-branes being present. We argue that during the inflationary phase the dominant contribution to the potential originates from the forces between the M5-branes. These arise from open membrane instantons stretching between pairs of M5-branes. In section 3 we describe the regime in moduli space where we can map the M5-brane dynamics to the dynamics of assisted inflation. During inflation the distances between the M5-branes, which represent the inflatons, will grow until the M5-branes coalesce with the boundaries via small instanton transitions. This will partly reheat the universe. Towards the end of inflation the orbifold size will start to grow, pushing the hidden gauge theory to strong coupling. The onset of gaugino condensation on the hidden boundary and the ensuing non-trivial  $H$ -flux will stabilize in particular the orbifold size. These additional contributions to the potential will terminate inflation and the implied stabilization of the orbifold modulus will contribute to the reheating,

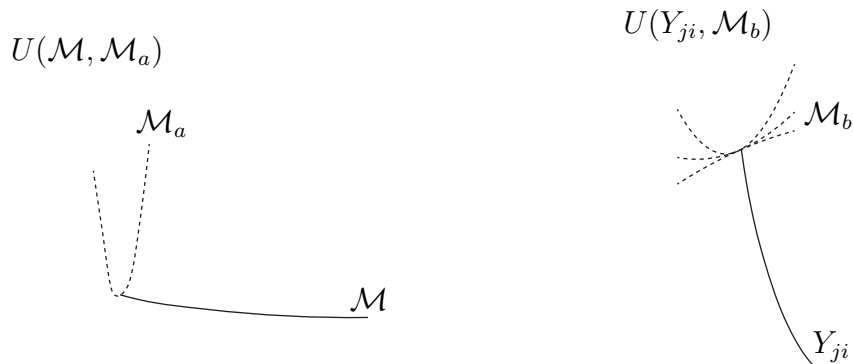


Figure 1: *The realization of new inflation (left figure) requires an extremely flat potential in the direction of the modulus  $\mathcal{M}$  playing the role of the inflaton. This ensures that the  $\mathcal{M}$  kinetic term is small and leads approximately to an exponential expansion  $a(t) \sim e^{Ht}$  where  $H \simeq \sqrt{U/3M_{Pl}^2}$ . Necessarily the potential in all other moduli directions  $\mathcal{M}_a$  has to be strongly curved upwards. In contrast, for the realization of assisted inflation (right figure) we use identical steeply decreasing exponential potentials for many moduli  $Y_{ji}$  which serve as inflatons. The outcome is a power-law inflation  $a(t) \sim t^{p(N)}$ . Since the potential in the  $Y_{ji}$  directions are the steepest directions available during inflation, the universe will follow their path even if the potential has a mild runaway in some of the remaining moduli directions  $\mathcal{M}_b$ .*

as well. In section 4 we describe in more detail the exit from inflation and derive the implications of our M-theory inflation proposal for the spectral index and the number of e-foldings. With a moderate number of M5-branes both of these quantities comply with their observationally derived values.

## 2 The Multi M5-Brane Potential

We are focussing in this work on embedding inflation into heterotic M-theory. As motivated in the introduction, our aim will be to show that assisted inflation can arise in M-theory. To this end we have to find a setup with several scalar fields, having each the same exponential potential. Let us therefore consider heterotic M-theory in the presence of  $N$  parallel M5-branes distributed along the  $\mathbf{S}^1/\mathbf{Z}_2$  orbifold interval. When compactified down to four dimensions on a six-manifold preserving  $N=1$  supersymmetry in four dimensions, the background is given through a warping of the six-manifold along the

$\mathbf{S}^1/\mathbf{Z}_2$  interval [20], [21]. In these solutions the six-manifold can either be a Calabi-Yau manifold or a more general non-Kähler manifold [22], [23], [24], the relevant warping for both cases will be the same. For simplicity, we will take a Calabi-Yau manifold henceforth. All  $N$  M5-branes will fill the four-dimensional non-compact spacetime and wrap the same holomorphic two-cycle  $\Sigma_2$  on the Calabi-Yau. To illustrate the mechanism in its simplest form, we will work with one Kähler modulus  $T$  only, hence take  $h^{1,1} = 1$ .

The effective four-dimensional N=1 supergravity theory is described in terms of the volume modulus  $S$  of the Calabi-Yau, the modulus associated to the length of the  $\mathbf{S}^1/\mathbf{Z}_2$  orbifold  $T$  and the M5-brane position chiral superfields  $Y_i$  (we are suppressing the gauge bundle moduli related to the  $E_8$  Yang-Mills gauge sectors)

$$S = \mathcal{V} + \mathcal{V}_{OM} \sum_{i=1}^N \left( \frac{x_i^{11}}{L} \right)^2 + i\sigma_S, \quad (2.1)$$

$$T = \mathcal{V}_{OM} + i\sigma_T, \quad (2.2)$$

$$Y_i = \mathcal{V}_{OM} \left( \frac{x_i^{11}}{L} \right) + i\sigma_i, \quad i = 1, \dots, N. \quad (2.3)$$

Here  $\mathcal{V}$  denotes the Calabi-Yau volume averaged over  $\mathbf{S}^1/\mathbf{Z}_2$  and  $\mathcal{V}_{OM}$  the averaged volume of an open membrane instanton wrapping  $\Sigma_2$  and stretching from one boundary to the other.  $L$  is the length of the  $\mathbf{S}^1/\mathbf{Z}_2$  interval and the position modulus of the  $i$ th M5-brane ranges over  $0 \leq x_i^{11} \leq L$ . The axions  $\sigma_S, \sigma_T, \sigma_i$  arise from various components of the three-form potential  $C$  of eleven-dimensional supergravity, see [25]. Note the correction to the real part of  $S$  which arises through the presence of the M5-branes [26], [18]. A similar correction arises in the Kähler-potential. A priori each M5-brane could wrap  $\beta_i$  times the basis two-cycle  $\Sigma_2$ . The above expressions assume, for simplicity, that all  $\beta_i$  are equal to one. Furthermore, there are  $h^{2,1}$  complex structure moduli  $Z^\alpha$ . Let us define in addition the real moduli<sup>7</sup>

$$s = S + \bar{S}, \quad t = T + \bar{T}, \quad y_i = Y_i + \bar{Y}_i, \quad (2.4)$$

$$y = \left( \sum_{i=1}^N y_i^2 \right)^{1/2}. \quad (2.5)$$

It will be convenient to further define

$$Q = s - \frac{y^2}{t}, \quad (2.6)$$

$$R = 3Q^2 - 2\frac{y^4}{t^2}, \quad (2.7)$$

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<sup>7</sup>Note the typographic difference between cosmic time  $t$  and the modulus  $t$ .

which will appear in the Kähler-potential and the matrix of its second derivatives. We will see shortly that in order to have a well-defined Kähler potential and a positive definite Kähler metric  $K_{I\bar{J}}$  of second derivatives, both  $Q$  and  $R$  have to be positive. The requirement of having  $R > 0$  is more stringent and imposes the following restriction on the  $s, t, y$  moduli

$$(3 - \sqrt{6})s > \frac{y^2}{t} . \quad (2.8)$$

The Kähler-potential for these moduli

$$K = K_{(S)} + K_{(T)} + K_{(Y)} + K_{(Z)}, \quad (2.9)$$

is given by [27], [26], [18]

$$K_{(S)} + K_{(Y)} = -\ln \left( S + \bar{S} - \frac{\sum_{i=1}^N (Y_i + \bar{Y}_i)^2}{T + \bar{T}} \right), \quad (2.10)$$

$$K_{(T)} = -\ln \left( \frac{d}{6} (T + \bar{T})^3 \right), \quad (2.11)$$

$$K_{(Z)} = -\ln \left( i \int_{CY} \Omega \wedge \bar{\Omega} \right), \quad (2.12)$$

with  $d$  the Calabi-Yau intersection number. Since, we can more succinctly write

$$K_{(S)} + K_{(Y)} = -\ln Q , \quad (2.13)$$

it is indeed clear that  $Q$  has to be positive

$$Q > 0 . \quad (2.14)$$

Another way to see this is to rewrite  $Q = 2\mathcal{V}$  and to note that in the backgrounds of [21] the average Calabi-Yau volume is always positive, as it should. Moreover, one finds that the determinant of the ensuing Kähler metric is (the indices  $I, J, \dots$  run over all complex moduli)

$$\det K_{I\bar{J}} = \frac{16R}{Q^{2N} t^6} \det G_{\alpha\bar{\beta}} , \quad (2.15)$$

with  $G_{\alpha\bar{\beta}}$  the metric on the complex structure  $Z^\alpha$  moduli space. Therefore, also  $R$  has to be positive

$$R > 0 , \quad (2.16)$$

to ensure a positive definite Kähler metric  $K_{I\bar{J}}$ .



Let us next discuss the relevant superpotential. During the epoch of inflation we will assume vanishing vacuum expectation values for charged matter fields. The corresponding trilinear Yukawa superpotential and higher-order perturbative boundary-boundary contributions [28] will therefore be absent. The remaining contributions to the superpotential will come from open membrane instantons wrapping each the same  $\Sigma_2$  on the Calabi-Yau, and stretching either between both boundaries (99), between two of the M5-branes (55), between the visible boundary and an M5-brane (95) or between an M5-brane and the hidden boundary (59)

$$W_{OM} = W_{99} + W_{55} + W_{95} + W_{59} . \quad (2.17)$$

These superpotentials are given by

$$W_{99} = h e^{-T}, \quad (2.18)$$

$$W_{95} = h \sum_{i=1}^N e^{-Y_i}, \quad (2.19)$$

$$W_{59} = h \sum_{i=1}^N e^{-(T-Y_i)}, \quad (2.20)$$

$$W_{55} = h \sum_{i < j} e^{-Y_{ji}}, \quad (2.21)$$

where

$$Y_{ji} = Y_j - Y_i, \quad (2.22)$$

describe the difference in location of the  $j$ th and the  $i$ th M5-brane. Usually, one would also consider gaugino condensation [29] on the hidden boundary [30]

$$W_{GC} = -C_H \mu^3 e^{-\frac{1}{C_H} f_h}, \quad f_h = S + \gamma_h T + \frac{\sum_i \gamma_i Y_i^2}{T}, \quad (2.23)$$

where  $C_H$  is the dual Coxeter number related to the hidden gauge group and  $\mu$  is determined in terms of the ultraviolet cut-off scale for the hidden gauge theory, see [3]. The parameters  $\gamma_h, \gamma_i$  are defined as

$$\gamma_{h,i} = \beta_{h,i} \frac{\pi L}{V_v} \left( \frac{\kappa}{4\pi} \right)^{2/3} \int_{\Sigma_2} \omega, \quad (2.24)$$

where  $\omega$  is the Kähler form of the Calabi-Yau and  $V_v$  the Calabi-Yau volume at the location of the visible boundary. All  $\beta_i = 1$ , as explained before, and the integers  $\beta_h$  are

obtained as expansion coefficients of the second Chern classes of the hidden boundary vector  $F_h$  and the Calabi-Yau's tangent bundle  $TX$

$$c_2(F_h) - \frac{1}{2}c_2(TX) = \beta_h[\Sigma_2] . \quad (2.25)$$

Similarly, one obtains the integer  $\beta_v$  from the corresponding visible boundary bundles. The anomaly cancelation equation demands

$$\beta_v + \beta_h + \sum_{i=1}^N \beta_i = \beta_v + \beta_h + N = 0 . \quad (2.26)$$

Via a perfect square structure of the M-theory action, gaugino condensation implies a non-vanishing NS-NS three-form field-strength  $H$  of type  $(3, 0) + (0, 3)$  on the hidden boundary [31]. Hence, once gaugino condensation sets in, a perturbative flux superpotential

$$W_H = \int_{CY_h} H \wedge \Omega, \quad (2.27)$$

integrated over the Calabi-Yau on the hidden boundary has to be taken into account as well.

It is important to recall under which geometrical conditions gaugino condensation on the hidden boundary has to be taken into account and when it should be omitted. The background geometry of heterotic M-theory [21] is warped along the  $\mathbf{S}^1/\mathbf{Z}_2$  exhibiting a singularity at some finite coordinate distance where the warp-factor vanishes. Phenomenological considerations [20], [21] make it desirable to place the hidden boundary right at this singularity, which implies a strongly coupled hidden gauge theory in which gaugino condensation sets in. Actually, the hidden boundary needs not be placed at this critical distance by hand, but can indeed be stabilized here through stabilization of the orbifold length modulus  $T$  [25], [3]. This situation describes successfully the particle phenomenology as we witness it today [32], [33] leading at the same time to dark matter candidates from hidden matter [34]. However, the situation in the early universe during the epoch of inflation might be different. In particular, it is conceivable that the orbifold size starts off at a subcritical value<sup>8</sup> and grows only gradually to its critical value towards the end of inflation. Indeed to achieve a stabilization at the critical orbifold size, the hidden  $E_8$  gauge group must already be broken down to a group with considerably smaller  $C_H$  [3], [34]. Therefore, if at the beginning of inflation, we assume an unbroken  $E_8$  on the

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<sup>8</sup>Subcritical means smaller than the critical orbifold size which would place the hidden boundary right on top of the singularity.

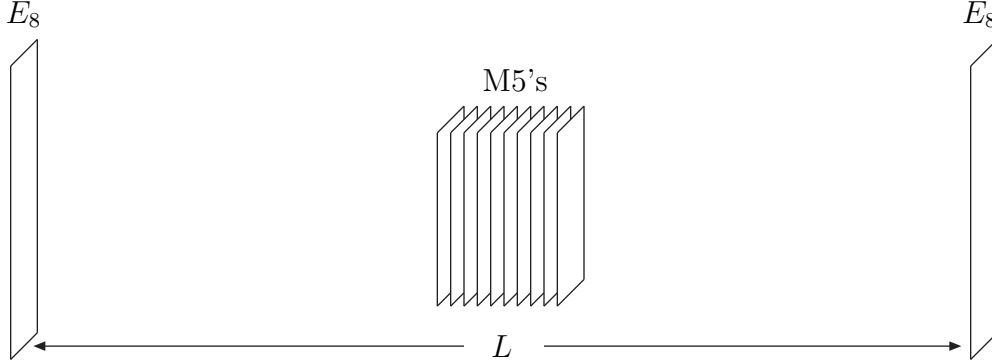


Figure 2: *At the beginning of the inflationary epoch, we assume all  $N$  M5-branes to be grouped around some common location on the  $\mathbf{S}^1/\mathbf{Z}_2$  interval such that the open membrane instanton interactions between the M5-branes dominate the potential.*

hidden boundary, we could stabilize the orbifold size only at subcritical value if the hidden gauge theory would remain strongly coupled. One has to notice, however, that when bringing the hidden boundary to subcritical distance, the hidden gauge theory will soon become perturbative since the Calabi-Yau volume on the hidden boundary quickly grows when the orbifold size shrinks [21]. More quantitatively, with a visible gauge coupling  $\alpha_v = 1/25$ , the hidden gauge coupling  $\alpha_h$  will be smaller than 1 (smaller than  $1/2$ ), if the orbifold size is less than 0.80 (less than 0.72) of its critical size. Under the assumption that at the beginning of inflation we start with such a subcritical orbifold size (we will see that this condition remains upright during inflation), it is therefore no longer justified to take gaugino condensation into account and we will consequently omit it. Necessarily, we then have to omit during inflation the  $H$ -flux superpotential induced by it, as well. Both contributions will, however, become important when the orbifold size grows towards the end of inflation to its critical size. We will therefore only take  $W_{OM}$  as our superpotential. Among these open membrane interactions we want to focus on those between the M5-branes, i.e. take for the superpotential

$$W = W_{55} . \quad (2.28)$$

This can easily be achieved by having initially all  $N$  M5-branes grouped together around some common generic site, not too close to either boundary, along the  $\mathbf{S}^1/\mathbf{Z}_2$  interval (see fig.2). Then the inter-boundary interaction  $W_{99}$  and the interactions between M5-branes and either boundary  $W_{95}, W_{59}$  can be neglected since the corresponding open membrane instantons have to stretch over longer distances.

The potential can then be calculated from the  $N=1$  supergravity expression for F-terms

contributions

$$U = M_{Pl}^4 e^K \left( \sum K^{\bar{I}J} D_{\bar{I}} \bar{W}_{55} D_J W_{55} - 3|W_{55}|^2 \right) , \quad (2.29)$$

which gives  $(D_i W_{55} \equiv \partial W_{55} / \partial Y_i + W_{55} \partial K / \partial Y_i)$

$$\begin{aligned} \frac{U}{M_{Pl}^4 e^K} &= G^{\bar{\alpha}\beta} D_{\bar{\alpha}} W_{55} D_{\beta} W_{55} + Qt \sum_{i,j=1}^N \left( \frac{1}{2} \delta_{ij} + \frac{Q}{Rt} y_i y_j \right) \bar{D}_i \bar{W}_{55} D_j W_{55} \\ &\quad + \left( \frac{3Q^2}{R} - \frac{2y^2}{Qt} \right) |W_{55}|^2 . \end{aligned} \quad (2.30)$$

The multiplying Kähler factor is

$$e^K = \frac{6}{(i \int \Omega \wedge \bar{\Omega}) Qt^3 d} . \quad (2.31)$$

Let us note that the second line of the potential comes from the terms

$$\sum K^{\bar{I}J} K_{\bar{I}} K_J |W_{55}|^2 , \quad (2.32)$$

with the sum running over all moduli, combined with the  $-3|W_{55}|^2$  term. One can check that  $3Q^2/R > 2y^2/Qt$ , i.e. the second line of the potential gives a positive contribution, if  $y < 1.83\sqrt{st}$ . Otherwise its contribution is negative. Now the moduli constraint (2.8) implies that  $y < \sqrt{3 - \sqrt{6}}\sqrt{st} = 0.74\sqrt{st}$ . Hence, the second line and therefore the whole potential will be *positive*. This is an important requisite for a derivation of power-law respectively assisted inflation.

## 3 Inflation from Multi M5-Brane Dynamics

### 3.1 The Inflationary Regime

We will now specify more precisely the regime in moduli space where inflation occurs, i.e. where we can map the dynamics of the interactions between the M5-branes to the dynamics leading to assisted inflation. First of all, we will impose the constraints

$$D_{\alpha} W_{55} = 0 , \quad (3.1)$$

$$D_i W_{55} = 0 . \quad (3.2)$$

Since we have just found that the potential is positive definite in these derivatives, abandoning these constraints would increase the potential energy and is thus disfavored on

dynamical grounds. These two constraints guarantee therefore a partial minimization of the potential. Let us now study their implications. The first constraint is equivalent to

$$\frac{\partial \ln h}{\partial Z^\alpha} = -\frac{\partial K_{(Z)}}{\partial Z^\alpha} . \quad (3.3)$$

This implies

$$h = i \int_{CY} \Omega \wedge \overline{\Omega} , \quad (3.4)$$

up to a  $Z^\alpha$  independent integration constant which we have set to zero. The  $h^{2,1}$  conditions (3.3) will fix the complex structure moduli.

Before discussing the implication of the second constraint, let us first derive an important upper bound on  $N$ . After applying the vanishing of the two Kähler-covariant derivatives, the potential  $U$  reads

$$\frac{d(i \int \Omega \wedge \overline{\Omega})}{6M_{Pl}^4} U = \left( \frac{3Q}{Rt^3} - \frac{2y^2}{Q^2 t^4} \right) |W_{55}|^2 . \quad (3.5)$$

If this potential should be mapped to an assisted inflation dynamics with the inflatons arising from the  $y_i$  differences, then we have to make sure that the inflatons appear only in the exponentials of  $W_{55}$ . In other words, during inflation the prefactor in brackets shouldn't depend on the inflatons and therefore not on  $y$ . This requirement can be met when the condition

$$Qt \gg y^2 , \quad (3.6)$$

is satisfied. It implies

$$Q \simeq s , \quad (3.7)$$

$$R \simeq 3Q^2 \simeq 3s^2 , \quad (3.8)$$

and also that  $3Q/Rt^3 \gg 2y^2/Q^2 t^4$ . Hence one can neglect the second term in the potential's prefactor and the potential becomes

$$\frac{d(i \int \Omega \wedge \overline{\Omega})}{6M_{Pl}^4} U = \frac{1}{st^3} |W_{55}|^2 , \quad (3.9)$$

with a  $y$  independent prefactor as desired.

It is interesting that the condition (3.6) amounts to an upper bound on  $N$ , the number of M5-branes. This narrows the range for  $N$  considerably, as we will see, and increases

the mechanism's predictability. Indeed, this bound is tighter than the bound (2.8) for a consistent supergravity, which becomes  $(3 - \sqrt{6})Qt > y^2$ . Let us add that on the other hand, we also know that with a too small  $N$  we won't be able to derive an assisted inflation mechanism. Hence the range for  $N$  giving inflation, is limited from above and also non-trivially from below.

After this preparation we can now address the geometrical meaning of the vanishing of the Kähler covariant derivative

$$D_i W_{55} = W_{55,i} + \frac{2y_i}{Qt} W_{55} = 0 . \quad (3.10)$$

To trust the effective supergravity description, we have to work, as always, in the regime where  $s, t$  are both considerably larger than one. We then find that  $Qt \simeq st \gg t > y_i$ . Hence, we can neglect the second term in the covariant derivative, reducing it to an ordinary partial derivative,  $D_i W_{55} \rightarrow W_{55,i}$ . To see most clearly what the vanishing of this derivative implies, let us concentrate from now on on the dominant nearest neighbor interactions between adjacent M5-branes, i.e. set

$$W_{55} = h \sum_{i=1}^{N-1} e^{-Y_{i+1,i}} . \quad (3.11)$$

The dominance of the nearest neighbor interactions is valid if the M5-branes are roughly equidistantly distributed. An exact equidistant distribution is however precisely what the vanishing of the derivative  $W_{55,i}$ , and therefore energy minimization, implies

$$Y_{i+1,i} \equiv \Delta Y . \quad (3.12)$$

All complex nearest-neighbor differences have the same value (see fig.2) and will be set equal to a common  $\Delta Y$  henceforth.

The equidistant M5-brane distribution, which makes the nearest-neighbor M5-brane interactions the dominant ones, allows us to derive for the potential  $U$  the following simple expression

$$\frac{Ud}{6M_{Pl}^4(i \int \Omega \wedge \overline{\Omega})} = \frac{(N-1)^2}{st^3} e^{-\Delta y} . \quad (3.13)$$

Here we have defined the real distance modulus

$$\Delta y = \Delta Y + \overline{\Delta Y} , \quad (3.14)$$

and employed the relation (3.4) to eliminate  $h$ .

### 3.2 Mapping the M5-Brane Dynamics to Assisted Inflation

Finally we have to transform the M5-brane position fields  $Y_i$  to fields whose kinetic terms are canonically normalized. The reason is that the assisted inflation dynamics is formulated in terms of canonically normalized scalar fields. The kinetic term for the  $Y_i$  is

$$S_{kin} = -M_{Pl}^2 \int d^4x \sqrt{-g} K_{i\bar{j}} \partial_\mu Y_i \partial^\mu \bar{Y}_{\bar{j}} , \quad (3.15)$$

with a summation over indices understood, where

$$K_{i\bar{j}} = \frac{4y_i y_j + 2Qt \delta_{ij}}{Q^2 t^2} . \quad (3.16)$$

It follows from (3.6) that  $Qt \gg y^2 = \sum y_i^2 > y_i y_j$ . Hence the first term in the numerator becomes negligible and we get  $K_{i\bar{j}} = 2\delta_{ij}/Qt$ . The canonically normalized complex fields are therefore  $M_{Pl} \sqrt{2/Qt} Y_i$  leading to the canonically normalized real M5-brane position and difference fields

$$\phi_i = \frac{2M_{Pl}}{\sqrt{Qt}} y_i, \quad \Delta\phi = \frac{2M_{Pl}}{\sqrt{Qt}} \Delta y . \quad (3.17)$$

We have just seen that a potential is only generated for the distance modulus between nearest neighbor M5-branes but not for their combined center-of-mass position. Let us therefore now switch from the  $N$  position fields  $\phi_i$  to the more adequate description in terms of the M5-brane center-of-mass field

$$\phi_{cm} = \frac{1}{N} (\phi_1 + \dots + \phi_N) , \quad (3.18)$$

and the difference field  $\Delta\phi$ . The relation between the two sets of fields is provided by the relation

$$\phi_i = \phi_{cm} + \left(i - \frac{N+1}{2}\right) \Delta\phi , \quad (3.19)$$

which is derived in the appendix. Since there is no potential for  $\phi_{cm}$ , its value will stay constant and its kinetic term vanishes. The sum of the  $\phi_i$  kinetic terms thus becomes

$$\frac{1}{2} \sum_{i=1}^N \partial_\mu \phi_i \partial^\mu \phi_i = \partial_\mu \Delta\phi \partial^\mu \Delta\phi \sum_{i=1}^N \left(i - \frac{N+1}{2}\right)^2 \quad (3.20)$$

$$= \frac{N(N^2 - 1)}{12} \partial_\mu \Delta\phi \partial^\mu \Delta\phi , \quad (3.21)$$

which requires us to perform a second rescaling. The canonically normalized difference field, denoted now by  $\varphi$ , with standard kinetic term  $(1/2)\partial_\mu\varphi\partial^\mu\varphi$  becomes

$$\varphi = \sqrt{\frac{N(N^2 - 1)}{6}}\Delta\phi = M_{Pl}\sqrt{\frac{2N(N^2 - 1)}{3Qt}}\Delta y . \quad (3.22)$$

We are now ready to map the dynamics of the single remaining difference field  $\varphi$  to the power-law inflation dynamics given in the introduction. For this, let us notice that written in terms of  $\varphi$ , we have found a potential (3.23)

$$U(\varphi) = \tilde{U}_0(N - 1)^2 e^{-\sqrt{\frac{3Qt}{2N(N^2 - 1)}}\frac{\varphi}{M_{Pl}}} , \quad (3.23)$$

where

$$\tilde{U}_0 = \frac{6M_{Pl}^4(i\int\Omega\wedge\overline{\Omega})}{st^3d} . \quad (3.24)$$

This positive valued prefactor can be regarded as being approximately constant throughout the inflationary period. The reason is that there are no steep potentials being present for  $s, t$  during the inflationary period where (3.23) is valid. The potential for  $t$  arising from  $W_{99}$  is strongly suppressed against the M5-brane potential arising from  $W_{55}$  and gaugino condensation. Gaugino condensation would deliver a steep potential for  $s$  but is absent as argued before. Therefore the size of the orbifold will stay approximately constant during the inflationary M5-brane evolution which we consider here.

In a spatially flat four-dimensional FRW universe we then have a Hubble parameter

$$H^2 = \frac{1}{3M_{Pl}^2}\left(U(\varphi) + \frac{1}{2}\dot{\varphi}^2\right) , \quad (3.25)$$

and the dynamics of  $\varphi$  is determined through

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dU}{d\varphi} = 0 . \quad (3.26)$$

This is precisely the dynamics which leads to power-law inflation [12] (see also [35]). To achieve the mapping to power-law inflation, (1.1), we merely need to set

$$p = \frac{4N(N^2 - 1)}{3Qt} , \quad (3.27)$$

$$U_0 = \tilde{U}_0(N - 1)^2 . \quad (3.28)$$



Hence, we end up with a power-law evolution for the scale-factor (1.2) and the solution (1.3) for  $\varphi$ . As discussed earlier, we need  $p > 1$  to obtain power-law inflation. Hence, we have to impose the further constraint

$$p > 1 \quad \Leftrightarrow \quad 4N(N^2 - 1) > 3Qt . \quad (3.29)$$

During the inflationary phase both  $Q$  and  $t$  stay approximately constant and  $y$  does not vary much since the M5-branes spread towards both boundaries. Therefore the two conditions (3.6) and (3.29) remain valid during inflation, if they have been fulfilled initially. Moreover, since during inflation the M5-brane nearest-neighbor distances grow in an equidistant way, also (3.2) remains satisfied. The same is of course true for (3.1). This is no surprise as we have seen that the vanishing of both Kähler covariant derivatives minimizes the energy and are thus dynamically selected. We can therefore conclude that all the conditions which are needed to generate inflation, do not break down during the inflationary process.

This is a very interesting result, as it shows that assisted inflation can be realized successfully in heterotic M-theory. The embedding of assisted inflation into string theory was explored previously in the type IIB context in [36]. There it was found that tree level potentials resulting from fluxes do not induce potentials that lead to inflation and it was speculated that instanton corrections may lead to such a potential. This is precisely what we have shown herein. Indeed type IIB brane-world models proposed in [37] have a two boundary setup very similar to the  $\mathbf{S}^1/\mathbf{Z}_2$  setup studied here. With the role of the M5-branes played by D5 or D7-branes it might thus be possible to transfer the M-theory inflation mechanism also to type IIB within this brane-world framework.

## 4 Exit from Inflation and Observational Results

So far we have demonstrated that inflation in M-theory can arise from multi M5-brane dynamics. The idea is to have initially all M5-branes rather close together such that their dynamics dominates the potential. In particular the boundary-boundary interactions  $W_{99}$  are negligible and the growth of the distances between adjacent M5-branes will be much more rapid than the growth of any other modulus. The ensuing dynamics can be mapped to power-law inflation, leading to a sustained period of inflation in the regime determined by the two constraints (3.29), (3.6)

$$\frac{4}{3}N(N^2 - 1) > Qt \gg y^2 , \quad (4.1)$$

The left inequality ensures  $p > 1$  and therefore guarantees inflation, whereas the right inequality led to the simple exponential potential (3.23) required for power-law inflation. From these constraints it is clear, that we wouldn't have obtained inflation if just one or too few M5-branes would have been present. In that case  $p$  would be much smaller than 1 and cannot give inflation. To obtain power-law inflation from a multitude of equal exponential potentials which are too steep to give inflation on their own, but will do so when considered together due to an increased Hubble friction, is the idea of assisted inflation [11]. It is thus an assisted inflation mechanism which we have derived here from M-theory.

Let us comment on the naturalness of the initial configuration of M5-branes. The positive potential (2.30) is in particular positive definite in the Kähler covariant derivatives. Therefore, setting  $D_i W_{55} = 0$  already minimized partially the energy and is therefore dynamically motivated. In the large volume regime, in which we are working, this condition became simply  $W_{55,i} = 0$ . When considering just nearest-neighbor open membrane interactions, the geometrical meaning of this equation was precisely that the M5-branes had to be equidistantly distributed. Therefore, among the many initial M5-brane distributions, the equidistant ones are dynamically favored. It remains to find a selection principle for the initial smallness of the common nearest-neighbor distance and the initial localization of the stack of M5-branes away from the boundaries. While the former issue constitutes a fine-tuning for the time being, it can be shown that the latter issue of having all M5-branes localized away from the boundaries can actually be relaxed to an initial equidistant configuration along the whole interval. This will be dealt with in a separate publication.

In the rest of this section we would like to address the two main observational implications and the exit from inflation. An important quantity predicted by an inflation model is the spectral index  $n$ . It determines the power-law spectrum of the primordial curvature perturbation  $\mathcal{P}_{\mathcal{R}}(k) \sim k^{n-1}$ . From this spectrum the spectrum of any other perturbation can be obtained by simple multiplication with the square of the appropriate transfer function [35]. For power-law inflation the spectral index  $n$  is given by

$$n = 1 - \frac{2}{p} . \quad (4.2)$$

Observations lead to the constraint [38]<sup>9</sup>

$$n = 0.98 \pm 0.02 , \quad (4.3)$$

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<sup>9</sup>We are grateful to R. Kallosh for bringing this reference to our attention.

which implies

$$p \simeq 100 . \quad (4.4)$$

We will now see that we can account for this observational constraint within the regime where our derivation is valid.

To this end, let us first make both constraints obtained so far concrete by adopting typical values  $\mathcal{V} = 341$ ,  $\mathcal{V}_{OM} = 7$  (cf. the values in table 1 of [3] for the relevant case of a hidden unbroken  $E_8$ ) and  $x_i^{11}/L = \mathcal{O}(1/2)$ . These imply  $s = 682 + 3.5N$ ,  $t = 14$ ,  $y^2 \simeq 49N$ . The constraints (4.1) then deliver the following bound on  $N$

$$19 < N \ll 195 . \quad (4.5)$$

With  $N$  in this regime we will obtain inflation. Let us next verify that the spectral index observational constraint indeed amounts to an  $N$  within this range. For this we evaluate (3.27) with the same  $\mathcal{V}$ ,  $\mathcal{V}_{OM}$  as before, which gives

$$p \simeq \left( \frac{N}{19.3} \right)^3 . \quad (4.6)$$

The spectral index constraint (4.4) then amounts to

$$N \simeq 89 , \quad (4.7)$$

which indeed lies in the above interval. Thus, without having to invoke extremely large values for  $N$ , we can account for the correct size of the spectral index  $n$ .

Before addressing the next important quantity, the number of e-foldings, it will be necessary to describe first how and when the exit from inflation occurs. The simple exponential potential which we have found, remains valid as long as the other contributions to the potential, gaugino condensation,  $H$ -flux, and the 99, 59, 95 open membrane instantons are absent or remain negligible. In particular this requires that the hidden gauge theory shouldn't be strongly coupled during inflation to avoid gaugino condensation to set in. This can be easily achieved by starting with a subcritical orbifold length at the beginning of inflation. Moreover, we have to make sure that towards the beginning the M5-branes have nearest neighbor distances which are much smaller than the distance between an M5-brane to either boundary (see fig.2). This ensures that it is indeed the 55 M5-brane sector which dominates the potential. What happens during the inflationary phase is that the coordinate distance

$$\Delta x(\mathbf{t}) \equiv x_{i+1}^{11}(\mathbf{t}) - x_i^{11}(\mathbf{t}) = \frac{L}{2\mathcal{V}_{OM}} \Delta y(\mathbf{t}) , \quad (4.8)$$

between adjacent M5-branes grows, whereas the Calabi-Yau volume and the orbifold size stay approximately constant (see fig.3). Once, however, the M5-brane distances have grown to a size comparable to the orbifold size  $L$  itself, the  $W_{99}$  open membrane instanton contribution will become of the same size as  $W_{55}$ . This additional exponential contribution to the potential will then cause the orbifold size to grow as well. This growth will however soon end. The reason is that when the orbifold size grows the hidden boundary Calabi-Yau volume shrinks, rendering the hidden gauge theory strongly coupled and setting off gaugino condensation. Once gaugino condensation is present it will counterbalance the expansion caused by the open membrane instantons and stabilize the orbifold modulus  $T$  as worked out in detail in [3].

Let us finally comment on  $W_{59}, W_{95}$ . Their biggest contribution will come from the two outermost M5-branes closest to the hidden resp. the visible boundary. Let us estimate when their contribution will equal  $W_{55}$ . Due to the symmetry of the problem it is enough to focus on the visible boundary side. With  $|W_{95}| \simeq |h|e^{-y_i/2}$  and  $|W_{55}| = |h|(N-1)e^{-\Delta y/2} \simeq |h|Ne^{-\Delta y/2}$ , setting both contributions equal, gives

$$y_i = \Delta y - 2 \ln N . \quad (4.9)$$

This is equivalent to  $x_i^{11}/L = \Delta x/L - (\ln N)/\mathcal{V}_{OM} > 0$ , where the last inequality guarantees a positive  $x_i^{11}/L > 0$ . With  $N \gtrsim 36$  (coming from the spectral index constraint) and a value  $\mathcal{V}_{OM} = 7$  as above, this says that only if  $\Delta x/L$  has grown to surpass

$$\Delta x/L > 0.5 , \quad (4.10)$$

will the  $W_{59}$  (and by symmetry also the  $W_{95}$ ) become of similar size to  $W_{55}$  and need to be considered. This is, however, close to the end of inflation and will therefore be neglected during the inflationary phase itself.

As an indicator for when inflation comes to an end, we can therefore use the distance between adjacent M5-branes. While at the start of inflation at time  $t_i$ , we have

$$\frac{\Delta x(t_i)}{L} \ll 1 \quad \Leftrightarrow \quad \Delta y(t_i) \ll t , \quad (4.11)$$

we find that inflation stops at a time  $t_f$ , when (see fig.3)

$$\frac{\Delta x(t_f)}{L} \gtrsim \frac{1}{2} \quad \Leftrightarrow \quad \Delta y(t_f) \gtrsim \frac{t}{2} . \quad (4.12)$$

The reheating will happen when the M5-branes coalesce with the visible boundary through small instanton transitions while an additional contribution to the reheating will come

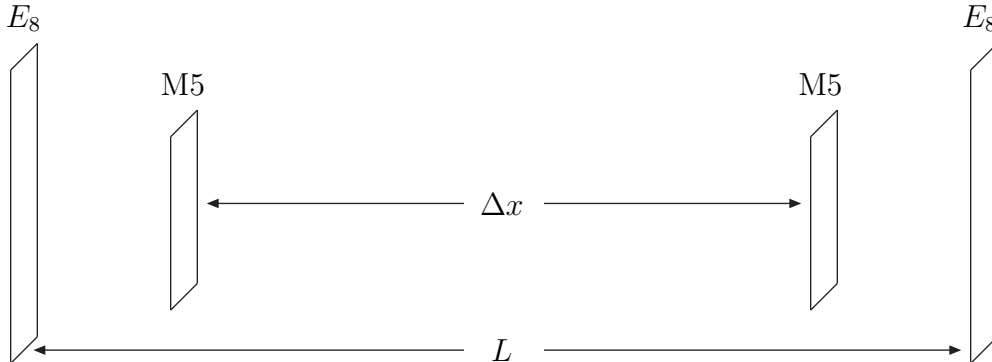


Figure 3: *Inflation comes to an end when the distance between adjacent M5-branes has grown to a size comparable to the orbifold size itself. At this stage most of the M5-branes have coalesced with the boundaries through small instanton transitions. This reheats partly the visible boundary and therefore our universe.*

from the stabilization of  $T$  towards the end of inflation. Small instanton phase transitions were initially discovered in [13] and studied in connection to heterotic M-theory in [14] (see also [39]). It was found that when an M5-brane disappears into the boundary and generates a singular torsion free sheaf, this sheaf which is referred to as a small instanton, can be smoothed out to a smooth holomorphic vector bundle by moving in its moduli space. This process changes the boundary's instanton vacuum. This change of topological data will generically alter the boundary's unbroken gauge group. For very specific initial topological data also chirality changes can be induced, changing the number of quark and lepton families on the visible boundary. In contrast to the  $SO(32)$  small instanton, which can be described in terms of some massless fields that appear at the singularity, it is believed that a non-trivial six-dimensional conformal field theory governs the  $E_8 \times E_8$  small instanton singularity<sup>10</sup> [40]. It would be interesting to explore whether cosmic strings as gauge theory solitons or the M-theory equivalent of the recently found cosmic superstrings of [41] could arise in this phase transition.

But let us now, after having given a criterium for the end of inflation, determine the number of e-foldings generated during inflation. This number is given by

$$N_e \equiv \ln \left( \frac{a(t_f)}{a(t_i)} \right) = p \ln \left( \frac{t_f}{t_i} \right), \quad (4.13)$$

which is usually assumed to lie between 50 and 60. Since the criterium (4.12) for the end of inflation is expressed in terms of the difference  $\Delta x(t)$ , we have to express cosmic time

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<sup>10</sup>We thank E. Witten for helpful comments.

$t$  in terms of  $\Delta x(t)$ . This is easily done using the explicit solution for  $\varphi(t)$  given in (1.3). The result is

$$\frac{t}{M_{Pl}} \sqrt{\frac{U_0}{p(3p-1)}} = e^{\frac{t\Delta x(t)}{2L}}. \quad (4.14)$$

The number of e-foldings can thus be expressed in terms of the geometrical M5-brane position difference as follows

$$N_e = \frac{tp}{2} \left( \frac{\Delta x(t_f)}{L} - \frac{\Delta x(t_i)}{L} \right) \simeq \frac{tp}{2} \frac{\Delta x(t_f)}{L} \gtrsim \frac{tp}{4}. \quad (4.15)$$

The second approximation uses the fact that, at the beginning of inflation,  $\Delta x$  was much smaller than at the end (4.11), while the last approximation uses (4.12).

To determine an actual value for  $N_e$ , let us adopt the same values for  $\mathcal{V}, \mathcal{V}_{OM}$  as before. Then by using (3.27) to express  $p$  in terms of  $N$ , we arrive at

$$N_e \simeq \left( \frac{N}{12.7} \right)^3. \quad (4.16)$$

With  $N \simeq 89$ , as required by the spectral index constraint, we obtain  $N_e \simeq 345$ . To comply with observation one needs at least 50-60 e-foldings. There is no upper bound on this number as everything what happens before the last 50-60 e-foldings will not be observable<sup>11</sup>. We can therefore conclude that both the spectral index and the number of e-foldings can be obtained in a realistic regime from our proposed mechanism for M-theory inflation.

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## A Conversion Formula

Let us here derive the conversion from the set of M5-brane position fields  $\phi_i$  to the set of center-of-mass and difference fields  $\phi_{cm}, \Delta\phi$ . The center-of-mass field is defined as

$$\phi_{cm} = \frac{1}{N}(\phi_1 + \dots + \phi_N) . \quad (\text{A.1})$$

Since the differences between all neighboring M5-branes are the same, i.e.  $\phi_{i+1,i} = \Delta\phi$ , we have for the individual M5-brane position fields the obvious expression

$$\phi_i = (i-1)\Delta\phi + \phi_1 . \quad (\text{A.2})$$

It remains to express  $\phi_1$  as a function of  $\phi_{cm}$  and  $\Delta\phi$ . This can be easily achieved by using

$$N\phi_{cm} = \sum_{i=1}^N \phi_i = \Delta\phi \sum_{i=0}^{N-1} i + N\phi_1 = \frac{N(N-1)}{2}\Delta\phi + N\phi_1 ,$$

from which we obtain the desired result

$$\phi_1 = \phi_{cm} - \frac{N-1}{2}\Delta\phi . \quad (\text{A.3})$$

Substituting this into (A.2), gives us finally the conversion from the position fields  $\phi_i$  to the center-of-mass and difference fields  $\phi_{cm}, \Delta\phi$

$$\phi_i = \phi_{cm} + \left(i - \frac{N+1}{2}\right)\Delta\phi . \quad (\text{A.4})$$

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